Decomposition methods for the numerical solution of multidimensional problems of anomalous diffusion

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Let $Q_T = G \times (0, T]$ be a cylinder with the base

$$G = \{x = (x_1, \dots, x_p)|,\ 0 < x_r < l_r,\ r = \overline{1, p}\},$$

and Γ is the boundary of G.

Consider the following equation of two-sided anomalous diffusion:

$$({}^{\mathrm{C}}D^{\beta}_{0+,t}u)(x,t) = Lu(x,t) + f(x,t), \tag{1}$$

$$u(x,0) = u_0(x), \quad x \in \overline{G}, \tag{2}$$

$$u(x,t)\big|_{\Gamma} = 0, \quad 0 \leqslant t \leqslant T, \tag{3}$$

where

$$L = \sum_{r=1}^{p} L_r, \qquad L_r = q_r D^{\alpha_r}_{0+,x_r} + (1 - q_r) D^{\alpha_r}_{l_r-,x_r}.$$

The derivatives with respect to space variables are left and right Riemann-Liouville fractional derivatives ($1 < \alpha_r < 2$):

$$(D^{\alpha_r}_{0+,x_r}u)(x,t) = \frac{1}{\Gamma(2-\alpha_r)} \frac{\partial^2}{\partial x_r^2} \int\limits_0^x \frac{u(\dots, x_{r-1}, \xi, x_{r+1}, \dots, t)}{(x_r - \xi)^{\alpha_r - 1}} \mathrm{d}\xi, \tag{4}$$

$$(D^{\alpha_r}_{l_r-,x_r}u)(x,t) = \frac{1}{\Gamma(2-\alpha_r)} \frac{\partial^2}{\partial x_r^2} \int\limits_{x_r}^{l_r} \frac{u(\dots, x_{r-1}, \xi, x_{r+1} \dots, t)}{(x^r - \xi)^{\alpha - 1}} \mathrm{d}\xi, \tag{5}$$

and Caputo fractional derivative with respect to time ($0 < \beta < 1$):

$$({}^{\mathrm{C}}D^{\beta}_{0+,t}u)(x,t) = \frac{1}{\Gamma(1-\beta)} \int\limits_0^t \frac{u'(x,\tau)\mathrm{d}\tau}{(t-\tau)^{\beta}}. \tag{6}$$

Outline of the speech

- Explicit and implicit finite difference schemes are considered.
- The decomposition method of scheme operator is proposed.
- Stability is proved
- Numerical experiment